

About Options Investors

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Abstract

Many attempts have been made to estimate market sentiment from options prices, none more well known than the Black Scholes model. Yet no options pricing model is without significant criticism. And no existing model is consistent with the characteristics of the market discussed in this paper.

Prior to developing a pricing model, it is important to delve deep into the market that is to be modeled. It is the author's belief that important characteristics of securities markets and how securities are priced are ignored by many pricing models. In particular, these often-ignored characteristics of securities markets that this paper emphasizes are:

1. Securities markets are not homogenous; investors have different perspectives.
2. Investors are risk averse.
3. Most investors sit on the sidelines, choosing not to invest in a particular asset; as a result, assets are priced by outliers in the market, not investors that are representative of the market.
4. Options prices are set by investors with different objectives; as a result, options prices cannot be used to understand the sentiment of investors participating in buying and selling options.

This paper attempts to make a compelling case that each of these market characteristics is true and relevant. The paper does not offer a new options pricing model but instead offers a foundation upon which a model more consistent with the market can be built. Developing such a model should be approached not by asking why investors buy or sell an option at a particular price, but by asking why that price does not align with the utility function of most investors.

Introduction

Options offer a tantalizing instrument through which to extract market sentiment. For any particular asset and maturity date, there may be hundreds of options. With as many data points priced based on the perceived value of a single asset at a single point in time, it would seem plausible that significant insight into the market's expectations and uncertainty could be extracted from options prices.

This is not a unique view. Academics have committed significant effort towards extracting such market sentiment from options prices since at least the 1960s [1,2], and in the 1970s it was believed that a significant breakthrough was found with the Black Scholes options pricing model [3]. However, this model-imposed constraints inconsistent with the market, such as constant expected volatility and a constant expected return of the underlying asset [4]; look no further than the bond market and the VIX index respectively to see that expected return and volatility are not constant. Additionally, the model is reliant on a "hedged" portfolio which, the authors propose, the market will

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price to have a risk-free rate of return; however, for an investor to believe they are fully hedging the portfolio, they must know the marginal change in the value of the option with regards to a marginal change in the underlying asset. Only the Black-Scholes model offers a means of calculating this value. Thus, a circular argument arises: if the market believes in the Black-Scholes model, then investors can make the required calculations to assemble the hedged portfolio and then price it to be risk free; in this case the model may be true. But if the market does not believe in the model, then the market will not have no faith that the hedged portfolio is risk-free and will price the portfolio to be risky, contradicting the Black Scholes model; in this case, the model is certainly false. Unlike a law of nature which is true regardless of whether mankind believes or not [1], the Black Scholes model is erroneous if the market does have full faith in it and can only be true if the market does have full faith in it. As the volatility smile implies the market does not have complete faith in the model (a market that believes in the Black Scholes model would price all options with the same implied volatility), and as the Black Scholes model relies on the market having complete faith in it to be true, we can only conclude that the model is erroneous [5]. If the market has any doubt in the model, the model is invalidated and the market's distrust in the model is justified. In the 1970s, the concept of a risk-neutral probability distribution was also discussed [6] and a methodology for estimating the risk-neutral probability distribution was proposed [4,7]. However, it is acknowledged that the risk-neutral probability distribution is not the market's actual perceived probability distribution function since the market is not risk-neutral; no method of transforming the risk-neutral probability distribution into the market's risk-averse probability distribution function has yet to be proposed and accepted. Further attempts have been made to measure securities risk premia which could then allow for an estimate of the market's risk-averse (true) probability distribution. The Recovery Theorem [8] attempts to extract the risk adjustment factors for options using a Markov chain and a transformation matrix. This could potentially solve the problem left by risk-neutral probabilities. However, the theory is reliant on three assumptions which have not been adequately justified. Another approach used a volatility index as a proxy for risk premia [9]. Neither has demonstrated significant acceptance by analysts or the market.

This paper does not propose a new options pricing model. Rather, it proposes the following key characteristics of options investors as a foundation upon which a future pricing model can be constructed:

1. The market is not homogenous. Investors have different perspectives.
2. Investors are risk averse.
3. Most investors sit on the sidelines, choosing not to invest in a particular asset; as a result, assets are priced by outliers in the market, not investors that are representative of the market.
4. Options prices are set by investors (purchasers and writers) with different objectives; as a result, options prices cannot be used to understand the sentiment of investors participating in buying and selling options.

As previous attempts to model options prices have disregarded key elements of the market, the author believes that it is important to begin by creating this foundation.

Methodology

This paper builds on raw observations, a utility model from behavioral finance, and theory to support the qualities of the market that this paper claims to be true. Publicly available data on S&P 500 forecasts is referenced to support claim 1, that the market is not homogenous. Existing literature and a utility model from behavioral finance are offered to support claim 2, that investors are risk averse. The same utility model is then applied to prove claim 3, that many investors sit on the sidelines; evidence from the market is then provided to support the claim that in fact most investors sit on the sidelines. Finally, logical reasoning is applied to a puzzling observation about options prices to support the claim that options investors are motivated for a variety of reasons, implying that options prices cannot be used to understand the sentiment of investors who are participating in the buying and selling of options.

Discussion

This paper attempts to make a compelling case that each of the above-mentioned market characteristics is true and relevant, setting a foundation for achieving deeper visibility into market sentiment from securities prices in general and options prices in particular.

Varied market perspectives: The paper recognizes that the market is made up of individuals each with a different expectation for the future performance of an asset. Within financial organizations, there is wide discrepancy in forecasts for the S&P 500. Table 1 presents the forecasts as of June 1, 2023 for the S&P 500 at the end of 2023.

Table 1: Financial organization forecasts for the S&P 500.

Organization	S&P 500 forecast
BNP Paribas	\$3,400
Société Générale	\$3,650
Barclays	\$3,725
Morgan Stanley	\$3,900
UBS	\$3,900
Capital Economics	\$3,900
Citigroup	\$4,000
Credit Suisse	\$4,050
Wells Fargo	\$4,100
JPMorgan	\$4,200
Jefferies	\$4,200
RBC Capital Market	\$4,250
Bank of America	\$4,300
Evercore ISI	\$4,450
Goldman Sachs	\$4,500
Deutsche Bank	\$4,500
BMO Capital Markets	\$4,550
Piper Sandler's Craig Johnson	\$4,625
Fundstrat Global Advisors	\$4,750
Mean	\$4,155
Standard Deviation	\$360

Source: Morningstar.com, MarketWatch [11].

Note: All forecasts are as of June 1, 2023 for the S&P 500's price at the end of 2023.

The wide range of forecasts implies that the market is not homogenous. Instead, the market should be viewed as heterogenous with at least two investor-specific variables-investor forecasts and investor-specific forecast uncertainty. Future security pricing models should account for how this heterogeneity affects options prices.

Investors are risk averse: Investors dislike losses more than they appreciate gains. Previous attempts to empirically measure risk aversion have provided empirical evidence that investors are risk averse [10]. However, risk aversion is rarely considered when modeling options pricing (it is more often considered when pricing stocks). Risk-neutral pricing models, which explicitly assume that investors do not have an aversion to risk, price assets by calculating the probability weighted sum of unadjusted potential future values without an adjustment to risk. For example, such a model would say that an asset that has a 50% chance of having a value of \$10 at time T and a 50% chance of having a value of \$12 should have a value of \$11 (discounted appropriately for time). However, in reality, risk aversion would lead many investors to believe \$11 is too expensive as the risk of losing \$1 would not be fully offset by the possibility of gaining \$1. The investor would weigh the risk of the \$1 loss more than the possibility of the \$1 gain. As a result, a risk averse investor, as I contend most investors are, would discount this hypothetical asset further, pricing it somewhere between \$10 and \$11; the asset then would have a positive expected return to compensate for the risk the investor is taking.

This is not unfamiliar to us. Stocks have a higher expected return than bonds, presumably because of their greater risk. If investors were risk neutral, bonds would be priced to have the same expected returns as stocks. However, such is rarely if ever the case. The lower volatility of bonds allows them to trade such that their expected returns are quite low as investors are willing to pay a premium for their low-risk nature. As the market as a whole prices lower risk assets in this way, we can conclude that the market as a whole is risk averse.

When models are constructed without consideration to market participant risk aversion, they simply ignore a key factor in how markets price assets. This paper attempts to account for the market's risk aversion using a Constant Absolute Risk Aversion (CARA) utility function to value gains and losses from an investment.

$$U(y) = \frac{1}{R}(1 - e^{-Ry}) \tag{1}$$

where $U(y)$ is the utility function, y is the change in an investor's financial wealth (i.e. the financial gain or profit due to the security's change in price), and R is the coefficient of absolute risk aversion (Barseghyan, 2018). A positive R value indicates an investor is risk averse while a negative R value implies an investor is risk seeking. For the balance of this paper, we assume investors are risk averse and R is greater than 0. The utility function has been calibrated for this paper such that $U(0) = 0$.

Note that other utility functions exist and could be used instead of a CARA utility function.

Most investors are on the sidelines: The first two points help us conclude that most investors find securities too expensive to buy and too cheap to sell, resulting in them doing neither, choosing to sit on the sidelines with regard to that specific security.

We begin by showing that for each investor there is a price below which they would be willing to buy a put and another higher price above which the investor would be willing to write the put. If the price falls between these prices, the investor will choose to remain on the sidelines.

Theorem: If a risk-averse investor perceived P to be the fair price at which to buy a put, the same investor would need to be paid more than P to sell the put.

Proof: Let X be the set of a put's potential (positive) payouts x_n . The utility of each payout is less than the value of the payout:

$$U(x_n) = \frac{1}{R}(1 - e^{-Rx_n}) < x_n \tag{2}$$

for $R > 0$. The above is the intention of a risk-averse model which discounts the values of positive payouts and accentuates the values of negative payouts.

If each payout x_n has a p_n probability of occurring, using the probability weighted sum of potential welfare gains, the maximum the investor will pay for the put is.

$$P_{\text{maxpurchaseprice}} = \sum_x p_n \frac{1}{R}(1 - e^{-Rx_n}) < \sum_x p_n x_n \tag{3}$$

Conversely, for a seller, each payout is an obligation. Thus, the utility of each payout is.

$$U(x_n) = \frac{1}{R}(1 - e^{(-R)(-x_n)}) < -x_n \tag{4}$$

Note that these are negative values so the magnitude of $U(x_n)$ is greater than x_n . The minimum the investor will accept to write the put is.

$$P_{\text{minwriteprice}} = -\sum p_n \frac{1}{R}(1 - e^{(-R)(-x_n)}) > -\sum p_n (-x_n) = \sum p_n x_n \tag{5}$$

Note that negative sign in the above equation in order to ensure the price is positive.

Thus, the minimum price the investor would accept to write the put is greater than the probability weighted sum of potential payouts while the maximum price the investor would accept to purchase the put is less than the probability weighted sum potential payouts.

QED: If the average investor perceived the price of the put to be low enough to purchase, then the average investor would find the price too low to write. Thus, there would be strong demand to purchase the put but weak demand to write it. Similarly, if the average investor perceived the price to be high enough to write, there would be strong demand to write the put but low demand to purchase it. To even out supply and demand, which must be perfectly balanced for options, it is reasonable to conclude that the average investor finds the price of a put to be too high to purchase but too low to write, creating low demand and low supply. As we have proposed that there are various market perspectives, there will be some small portion of the market that finds the put low enough to purchase and a similarly small portion of the market that finds the put high enough to write, but many, if not most, investors would find the put too expensive to buy and too cheap to write.

This would apply to all securities, not just options. The evidence supports the proposition that most investors find securities too high to purchase and too low to short. In 2012, SigFig es-

timated that nearly 17% of all individual investors in the United States owned Apple stock [12]. Apple was the most widely held stock at the time; 6% of investors held Google stock and on average 4% of investors owned each of the stocks in the Dow Jones Industrial Average. One way to view this is that it is remarkable that Apple was able to convince 17% of investors in the United States to invest in its stock. But this figure also means that 83% of investors did not want to invest in Apple stock. An even larger percentage of investors did not short the stock. The majority of investors found Apple stock to be both too expensive to buy and too cheap to short. This point deserves reiteration; most investors considered the stock to be simultaneously too expensive to bet on and too cheap to bet against.

The observation here is that there is no single price above which investors will short and below which investors will buy. The space of investors is made up of three regions-buyers, sellers, and sideliners- with sideliners being the largest group. Any model of investor behavior must recognize that at any time, most investors' utility functions lead them to choose neither to buy nor to sell a stock. Stock prices are being set by the most optimistic (purchasers) and the least optimistic (short sellers) portions of the market; thus, the price of an asset does not reflect the sentiment of the market as a whole, as at all times the majority of the market would find the prevailing price of the asset to be incorrectly priced.

Take the Black-Scholes pricing model for example. The claim that an option's price is based on the market's implied volatility is incorrect. Option prices are not priced by the majority of the market, they are priced by a minority of the market. Options traders are less common than stock traders; it is the exceptional investor which participates in the pricing of options, not the average investor.

Options prices cannot be used to understand market sentiment of participating investors: A puzzle the author struggled with for some time was why the market would price an at-the-money put the same as an at-the-money call. As most securities go up in time, it would seem intuitive that an at-the-money call would be more valuable than an at the money put and thus would be priced higher by the market. Figure 1, which assumes a normal probability distribution of the underlying asset's fu-

ture price, depicts the difference in the probability that an at-the-money put will expire in-the-money versus the probability that an at-the-money call will expire in-the-money.

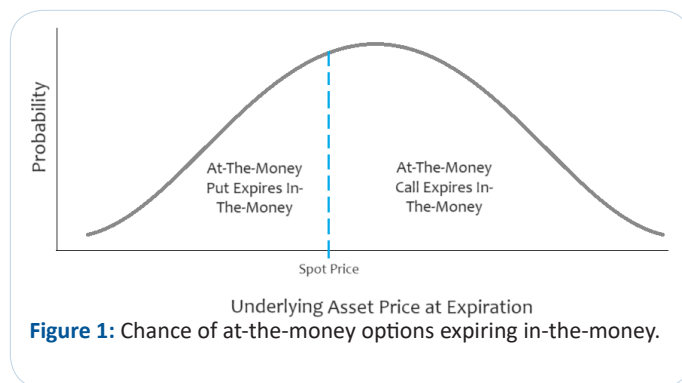


Figure 1: Chance of at-the-money options expiring in-the-money.

If the put has a lower expected value at expiration, the market should price the put lower than it prices the call. Yet this would violate put-call parity which requires at-the-money puts and calls to be the same price (with an adjustment for the risk-free rate). Otherwise, if the put was cheaper than the call, an easy arbitrage opportunity would be to short the call, buy the put, and short the underlying. The collective portfolio would generate a small risk-free cash flow to the investor that would represent an arbitrage opportunity.

So how can we rectify these two viewpoints: a) that an at-the-money call should be priced higher than an at-the-money put, and b) that an at-the-money call and put should have approximately the same price? of course, the answer is that while some market participants are purchasing these assets based on their future potential payouts, others are identifying violations of put-call parity which offer arbitrage opportunities. In theory, absent arbitrage investment, an at-the-money put would be priced below an at-the-money call because of the expectation that the underlying asset will go up more often than down (it should be noted that this is true of corporate stocks but is not necessarily true of commodities, foreign exchange, or volatility). Thus, the arbitrage traders must be pushing up the price of puts and pushing down the price of calls. Table 2 below, which categorizes investors active in purchasing or writing options, would be consistent with this theory.

Table 1: Investor incentives for purchasing or writing options.

	Calls	Puts
Write	Investors who believe the price of the underlying asset will not rise significantly.	Investors who believe the price of the underlying asset will not fall significantly. Arbitrage investors who are purchasing calls, selling puts, and shorting the underlying to create an arbitrage opportunity.
Purchase	Investors who believe the price of the underlying asset will rise significantly. Arbitrage investors who are purchasing calls, selling puts, and shorting the underlying to create an arbitrage opportunity.	Investors who believe the price of the underlying asset will fall significantly.

Arbitrage traders create excess demand for puts and excess supply of calls which skews options prices. The result is that the price of both calls and puts shift to price-points different than what they would be if investors were solely concerned about the expected value of the option. As there must be as many sellers as buyers with options, the result is that only a subset of put purchasers are buying because they believe the put to be attractively priced for its potential payout in the instance of a price decline and only a subset of call sellers are selling because they believe the call to be attractively priced based on its poten-

tial liabilities in the instance of a price rise. If there are X number of investors selling a put because they believe it is attractive for selling, there are fewer than X number of investors buying the put because they believe it is attractive for buying. Similarly, if there are X number of investors selling a call because they believe it is attractive for selling, there are more than X number of investors buying the call because they believe it is attractive for buying. While the number of buyers and sellers are equal, the number of buyers and sellers investing because of an option's potential future value are not equal.

As a result of this supply-demand shift from arbitrage investors, the price of options is skewed from the price point that would reflect the market's sentiment on the underlying asset. We cannot back out investor sentiment from options prices without first deriving some clever way of adjusting for the price skew resulting from arbitrage investors. As investors are participating in options for different reasons, options prices cannot be assumed to reflect broad investor sentiment of the underlying asset.

While not all investors who do buy or write options have the same motivation, investors who do not invest have more in common. Each investor who does not invest has assessed that a) there is no arbitrage opportunity and b) the option is both too expensive to buy and too cheap to write. Options prices tell us more about investors that are on the sidelines than they do about investors who are buying or selling options.

Conclusion

Investors are diverse, with unique sentiment on the future value of securities. It is because of this heterogeneity that we see individual investors make different investment decisions. Yet, for any asset, investors who buy and sell are a small minority of the market. With options specifically, not all purchasers and writers are engaged in purchasing and selling based on the security's future payouts. Options pricing models which focus on why investors purchase or write options will be incumbered by these facts.

Thus, it will be more fruitful to attempt to estimate market expectations by understanding the investor distribution; understanding this distribution should focus on evaluating the investors who do not invest rather than investors who do invest. Future options models may benefit from incorporating the above-discussed characteristics of the market.

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